Building Elementary Math Fluency Starter Kit



Christina Tondevold
TheRecoveringTraditionalist

Imagine how it will feel when your students can approach a math problem without getting frustrated. The way they think about math and numbers is a way that will help them solve any problem, even as they progress into different grade levels.

Not only will they have a solid base of mathematical knowledge, but you did it without using worksheets and timed tests.

Instead, you have activities that you and your students enjoy, that build important mathematical ideas, and don't take you forever to plan and prepare (i.e. you will NOT be waiting for the copier or the laminator anymore).

Instead, you'll be able to hang out with your family at night knowing that you have amazing plans for math the next day that build your students' math understanding and get students talking & thinking about math in ways you've never experienced before.

Can you even imagine that?

With a focus on flexibility you'll be starting with the foundation of number knowledge that will help your students build connections to all other areas of math. You'll be able to take activities straight from your book and modify them to help build math minds and not just create calculators.

Getting Started Building Math Fluency

To help you kickstart your building math fluency journey, there are three important decisions you need to make. The good news? Once you have all the details (which I'm going to give you in this Starter Kit) they are pretty easy decisions to make!

Decision #1: Is flexibility in math important?

Decision #2: How should you build fluency with math facts?

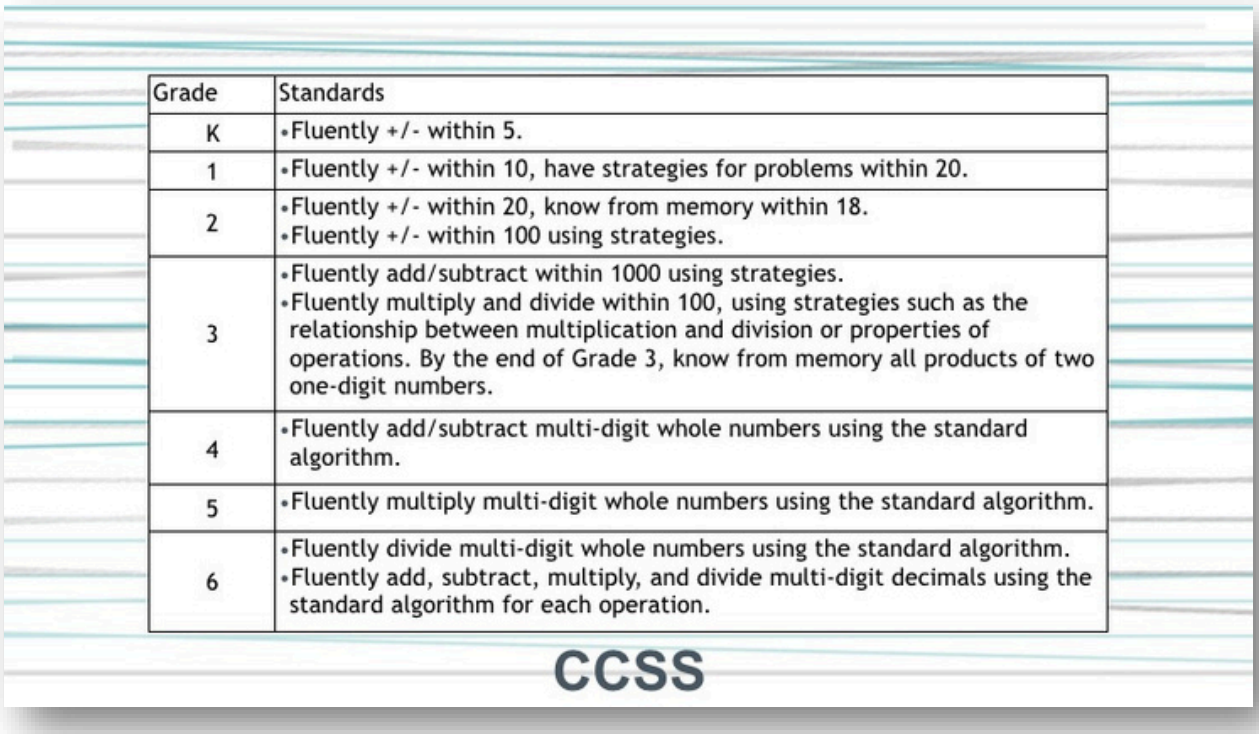
Decision #3: How can you arrange your math time to make room for building math fluency? (This one is extra fun to explore, so we'll keep it for the final decision!)

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Decision #1

Is Flexibility inMathImportant?

Math fluency is very important buttoo many people associate fluency with memorizing. Fluency does not mean memorize. It is so much more than just getting a correct answer and one of the biggest pieces to fluency is being flexible. Let's start off with looking at what fluency actually is and where it shows up in the standards.



Grade	Standards
K	•Fluently +/- within 5.
1	•Fluently +/- within 10, have strategies for problems within 20.
2	•Fluently +/- within 20, know from memory within 18. •Fluently +/- within 100 using strategies.
3	•Fluently add/subtract within 1000 using strategies. •Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
4	•Fluently add/subtract multi-digit whole numbers using the standard algorithm.
5	•Fluently multiply multi-digit whole numbers using the standard algorithm.
6	•Fluently divide multi-digit whole numbers using the standard algorithm. •Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

CCSS

The term fluently (or 'with fluency') shows up in the Common Core State Standards (CCSS) in every grade from Kindergarten through 6th Grade. I know that not everyone uses the CCSS, so check your own state standards but I'm sure they will be very similar if not the same.

So with every grade having some sort of 'fluency' standard, and even a couple that say 'know from memory,' it's no wonder we are all concerned about it. But 'know from memory' and 'memorize' are not the same thing. When you are truly fluent in something you know it from memory but that isn't the same as memorizing something.

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When the CCSS came out, there was a series of documents that came along with them called the Progressions. In the K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking, on pages 18-19 they define what they mean by the term 'fluent.'

*"The word **fluent** is used in the Standards to mean 'fast and accurate.' Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., 'adding 0 yields the same number'), and knowing some answers from the use of strategies....this is not a matter of instilling facts divorced from their meanings, but rather as an outcome of a multi-year process that heavily involves the interplay of practice and reasoning."*

I like this description of fluent except I believe they need to add in one more word to the first sentence: *The word fluent is used in the Standards to mean 'fast, accurate, and **flexible**.'* This three-word description of fluency comes from Susan Jo Russell's article about Computational Fluency in 2000.

The idea of being flexible is described in the Progression document where they say it "involves a mixture of...". They describe what being flexible looks like but they don't use the term 'flexible.' Their description says kids need to have various strategies to get their answer, which is what Russell described in her article.

What are the strategies?

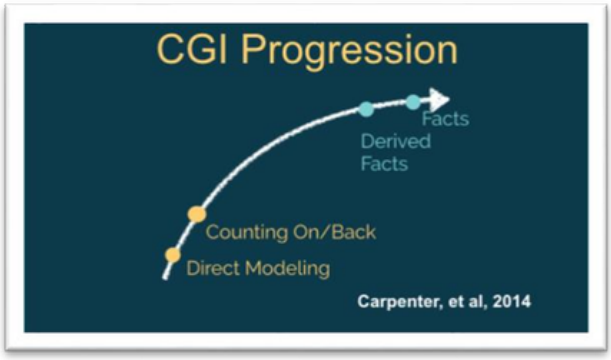
This is a *starter* kit so to help you get started we are going to look at the different strategies kids might have for addition and multiplication. We are going to explore all the different strategies, but here is my disclaimer: DO NOT teach your students all of these.

"Strategies Aren't Taught, They Are Caught."

Ilike to say that Strategies Aren't Taught, They Are Caught. We will go into more detail about how to help kids 'catch' these strategies in section two of this kit. For now, I just want to expose you to these different strategies so you are aware of ways your students might solve addition and multiplication problems so that you can make the decision about how important fluency is to you.

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Carpenter, et. al (1999) described these stages kids go through as they are developing fluency with operations:



Kids startoffwith**DirectModeling**wheretheymodelexactlywhatthe problem says. For 8 + 7 you will see a child count out 8, count out 7, and then gobackandrecountthemall.For6x9youwillseeachilddrawor model 6 groups of 9 and then count each item in the groups to determine the total.

When kids hit the **Counting On/Back** stage they don't need to model every part of the problem. For 8 + 7 a child will 'hold' the 8 in their mind and then count on the more 7. For 6 x 9, you might see a child draw or model 6 groups of 9 but then they count the groups of 9 (9, 18, 27…) instead of counting each item in the groups one-by-one. The examples I give here are for addition and multiplication so they are all "counting on", but when kids get into subtraction and division you will see them do the "counting back."

Most of the time once kids get into the Counting On/Back stage, we jump into having them practice & memorize so they can get to the **Fact** stage. This is the stage where a child just instantly knows it. In this stage, children might show that they are "fast and accurate" but the problem with jumping to this stage is that if a child doesn't just instantly know it, they fall back downtotheCountingstage.

Flexibility, the third part of fluency, shows up when we help kids progress into the **DerivedFact**stagebeforetheygototheFactstage.Forkidswho are solving8+7youmighthearthemsaysomethinglike*"7+7is14and1 more makes15."*For6x9,youmighthearakidsay*"3x9is27,sojust double itand6x9is27+27whichis54."*

That wasjustoneexampleforeachofthoseproblems,therearemanymore ways thatkidssolvethem.Tobetrulyfluent(fast,accurate,andflexible) kids needtobeoperatingintheDerivedFactandFactstages.Ifwefocus

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on helping kids memorize and drill them on their facts, they don't develop flexibility.

However, when we focus on helping kids build their flexibility, the by-product is that they become fast and accurate while also building their flexibility. The power isn't just in helping kids become fluent with their facts, but the real power is that when they build their flexibility with these small quantities it transfers to all kinds of quantities. As kids start working with problems like 399 + 456, they don't just know that problem as a fact and we also don't want them solving the problem with a counting strategy. Instead we want them using derived fact strategies.

We will look at the strategies for addition and multiplication with single-digits in Part 2 of this kit, but for now I want to look at the addition strategies using multi-digit numbers and then show you how one of those strategies progresses from single-digit operations into operating with fractions.

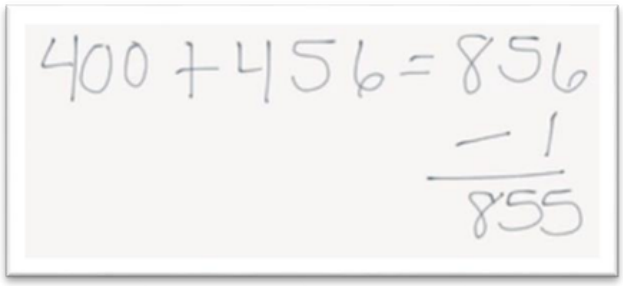
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5 Types of Addition Strategies

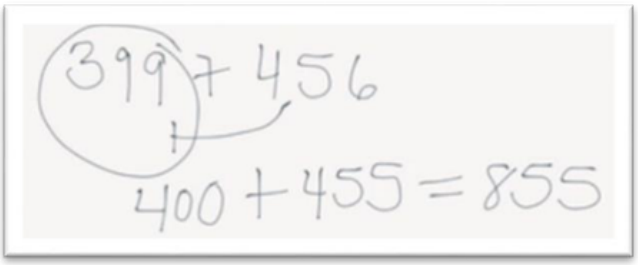
There are five strategies kids can use to solve Addition problems. Some kids might combine these but these are the five main strategies.

Let's look at solving 399 + 456 using these five strategies.

Compensating - is when you do something you're not supposed to do and you need to compensate for it. So with the 399, this is when you will see kids say, "Well that's almost 400 so if I just add 400 to 456 that would be 856." But they weren't supposed to add 400, so they need to compensate for what they did. That's why they subtract one at the end.

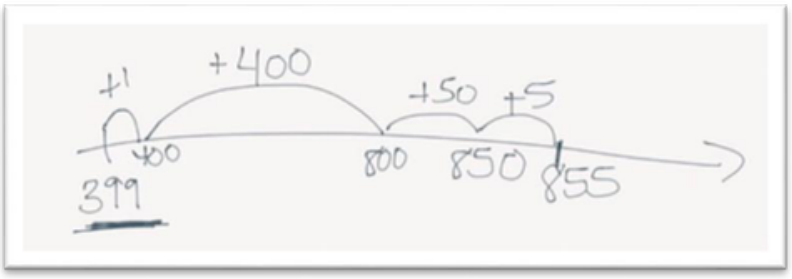


Give & Take - is really similar to compensating. You will see this in a kid who wants to make the 399 a 400. But instead of just rounding it to the 400, they decide to take 1 from the 456 and give it to the 399 so that they have 400. Then they are left with 455 and 400. So they can get directly to their answer after they have done this adjustment.

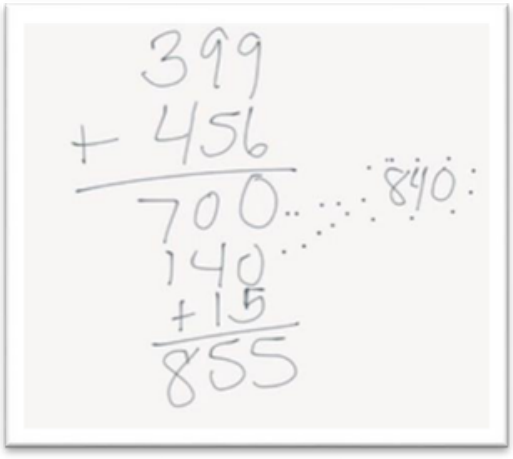


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Decomposing - is when kids break a number apart into chunks that make sense for them to add. It's just like decomposing in science, it's breaking something down into smaller pieces. So when adding 456, you might see a kid who chunks it out into 1+400+50+5 to make it easier to add 456.

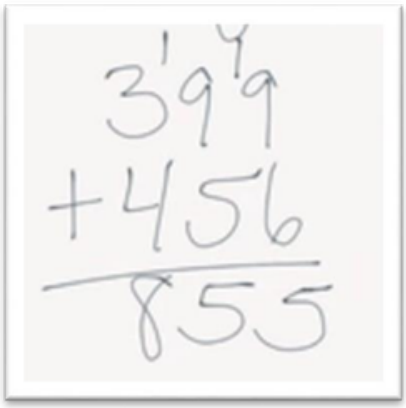


Like Values - is when kids will add the same values together. They add the ones with ones, tens with tens, and so on. So when adding 399 and 456, you might see a kid who adds the 300 and 400, then the 90 and 50, then the 9 and 6, and then puts those together to get their answer.



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Traditional Algorithm - is what most likely you learned when you were in school. You start with the ones and if there is more than ten you carry over a 'one' to the tens place. However, the algorithm itself is devoid of place value and only requires kids just "know their facts." If kids know their single-digit addition facts then this algorithm is easy because there is no strategy involved. That's why people like it so much is that it's very straight forward and works the same every single time.



The best part about these strategies is that we DO NOT directly teach them to students. It isn't your job as a teacher to show students all five ways to solve addition and make them solve one problem five ways. I'm showing them all to you here just so you can better understand the five strategies. Research shows that kids will naturally develop these strategies IF we focus on building their number sense and place value understandings.

"Itisn't yourjob asateacher toshow students allfive ways to solve addition and make them solve one problem five ways."

If we try to teach all these strategies to kids they are just learning more and more rules & procedures instead of thinking flexibly with numbers. If kids don't have flexibility, all they have is a procedure to follow. Then when they need to add some other types of numbers, they will need new rules and procedures to follow.

For example, when adding whole numbers we tell kids to "line up the numbers to the right," but then when they get into decimals we tell them to

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forget that and now they need to "line up the decimals." Instead, if kids are flexible thinkers with numbers the strategies they have for whole numbers still apply as they go into different types of numbers.

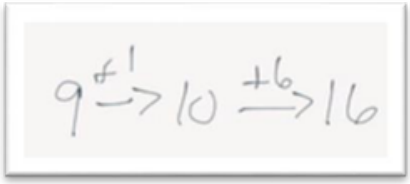
Let's take a look at just one strategy, Decomposing, and how it starts with single-digit numbers and progresses into working with all types of numbers.

How Flexible Thinking Progresses Through Elementary Math

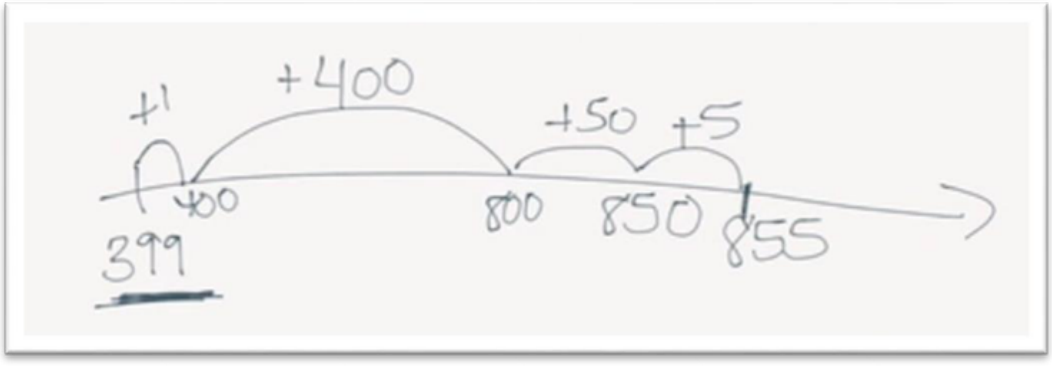
One of the most powerful things aboutbuilding your students' flexibility with numbers is that the flexibility laststhroughout all of the mathematics they are doing. We've all had those timeswhere kids are not able to transfer knowledge they seem to have inone area in math to another. Usually this is because they are seeing math as isolated rules and procedures instead of connections between numbers.

Let's take a look at how students who are flexible thinkers and use the Decomposing strategy progress throughout elementary math. Below are examples of using the Decomposing strategy on single digit addition, multi-digit addition, elapsed time, adding money, and adding fractions when measuring.

9 + 7

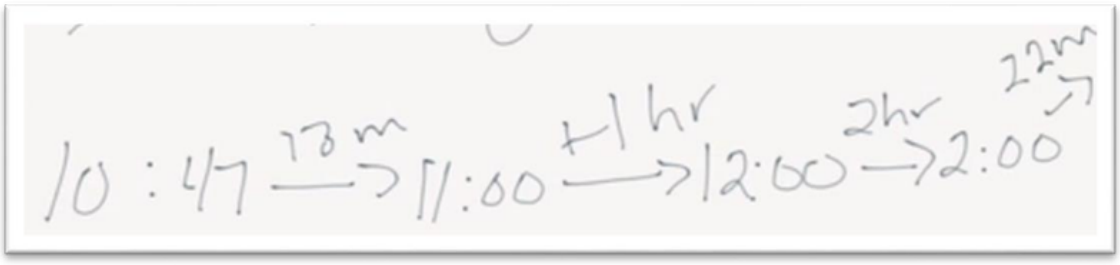


399 + 456

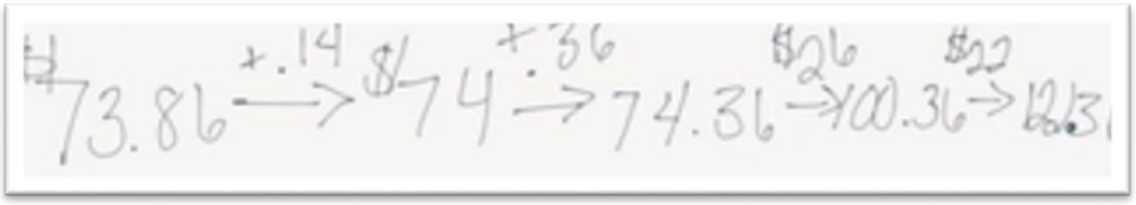


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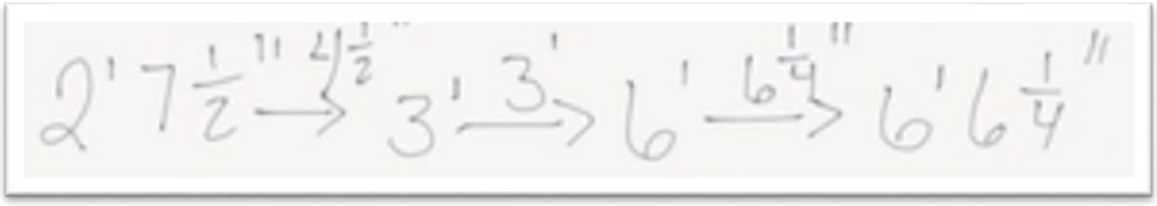
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A reminder that these strategies are not things you directly teach. Kidswill naturally do these strategies when they look at the numbers in the problem and they see relationships among the numbers. However, they do needa sense of numbers to help them come up with those strategies. The focusof fluency instruction needs to be on helping kids see those relationships instead of focusing on memorization of rules and procedures.

Kids will naturally invent their own strategies if we allow them to. Although you may not have grown up thinking like this, I bet if you open up the belief in your own mind that your students can do it, you’ll start to see more and more kids who are actually doing it.

So, you have a choice. You can choose to think that kids can’t invent these strategies on their own, and so it’s your job to just teach them.

Or you can choose to believe that students will invent these strategies, with guidance from you by allowing them to experience concepts around

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numbers, building their number sense, building their place value, and then help guide them through their invention of these strategies.

Now kids don't just magically wake up and say, "Oh, I see how to do this." It is tied to how they are thinking about single digit numbers, the number sense and experiences that they've had that help determine whether or not they will invent these strategies.

So, are you ready to focus on building students' flexibility of numbers so that they can invent these strategies on their own?

If so, let's dig into building fluency with facts in section 2!

3 Things to Help You Start Building Flexibility in Students

#1: Assess your Students' Fluency

When you hear that word 'assess' we immediately think 'test.' Assessing students can be done in a lot more ways than through a test. In fact most tests only assess two of the three parts of fluency: accuracy and efficiency. Most tests look to see how many answers kids can get correctly in a certain amount of time. That does not assess their fluency. You need to also be looking for their flexibility.

How to put this into action: A "quick" way to assess your students' fluency is to ask them how they solved a problem. I put "quick" in quotation marks because to sit down with each of your students one-on-one and ask them to explain their thinking can take a really long time. That's not what I'm suggesting. Instead make a point as kids are working on problems to ask a few students each day to explain their thinking on a problem. Make sure it is a problem they would be thinking flexibly about. For example, you can give a 1st grader 6+7 and they should be able to give you a way they figured it out (Hint: if they say *"I just know it's 13,"* then say, *"That's awesome! If you had a friend who didn't know, do you have a way you could explain it to them?"*). Now, if you gave that same problem, 6 + 7, to a 5th grader and asked them how they get the answer they may look at you funny thinking it's a trick question, so you might want to give something like 345 + 297. However, I did just ask my own daughter, who happens to be a 5th grader, how she figures out 6+7. She instantly said it's 13, but she said *"I take 3 from the 6, add it to the 7 so now I have 10+3 which is 13."*

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#2: Start with their 'basic' facts

How kids think about more advanced problems stems from how they think about the 'basic' facts. If they learned their facts through memorization only, they won't have thinking strategies for more advanced problems. Even kids who seem to know their basic facts can benefit from spending time going back and investigating relationships around those. Then you can help them make connections between their basic facts and the problems they are currently working on.

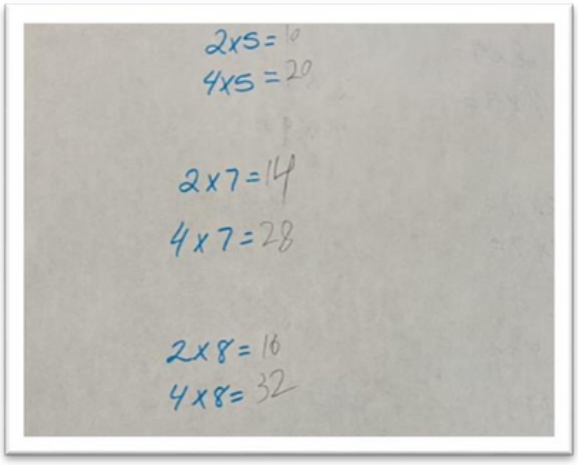
How to put this into action: We go more in-depth about how to build that flexibility for basic facts in the next section of this kit, but once you have spent time building that flexibility do activities that help kids see the connections between those basic facts and the problems they are solving now. This can be done by crafting a string of problems to help kids see how they solve one problem can help them solve another, more advanced, problem. After I gave my 5th grade daughter the 6+7 problem I asked her how she would solve 345+297. She quickly said "Take 3 from 345, then add it to 297, which makes 300. Now I have 300+342=642." So then I asked her 1679+2895. Can you guess her response? Take a moment based upon what she's done already, what would you expect her to do? That's the power of these strings and the power of connections. Her response was *"Take 105 from 1679, add it to 2895. Now we have 3000 + 1574 which equals 4574."* It all starts with helping kids build these strategies for their basic facts and then helping them connect it to all types of problems.

#3: Provide experiences that build number sense

Number sense is a compilation of relationships that build a 'sense of numbers.' There are 8 number sense concepts that I go into in my Flexibility Formula Course. It's more than what I can put in this paragraph, and we do talk about 4 of the concepts in section 2 of this kit, but the biggest thing you can do that builds number sense is asking students to look at how numbers and/or problems relate to each other. Just like I mentioned in #2 above, create strings of problems that help kids see connections. If they don't see connections easily, try just having kids look at numbers, pictures of objects, or problems and ask them "What do you notice? What do you wonder?" or "What's the same? What's different?"

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How to put this into action: We will go into a few of my favorite things to do that build number sense and flexibility in Part 3 of this kit, but a quick way to get started is take a look at the tasks you are giving to kids and if they are isolated, try giving related problem sets. For example, instead of giving kids the problem 2 x 5, give them 2 x 5 and 4 x 5. Then give them 2x7 and 4x7, 2x8 and 4x8. Spend time looking at how the two problems relate to each other, how the products & factors relate to each other. Some kids will see the connection right away: the product is double each time, but some kids won't. Through discussion and lots of repetition of doing problem strings, kids will naturally start to look for and use relationships they are seeing.



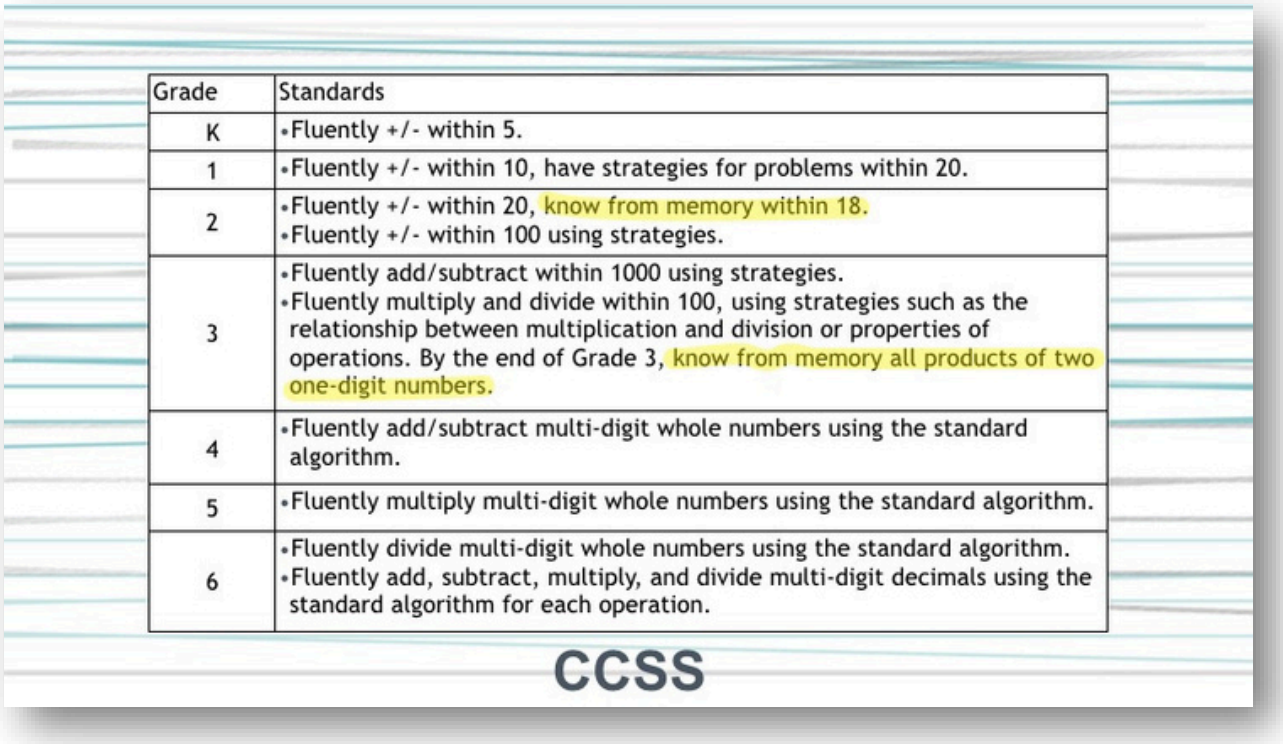
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Decision #2

How Should YouBuildFluencyWithMathFacts?

Idon't think I've ever come across a person who thought kids shouldn't know their math facts. The difference in opinions comes from HOW we help kids come to know their math facts.

First, let's get clear on what is meant by math facts. Some people also refer to them as the basic facts. Typically, it is the operations (add, subtract, multiply, and divide) done with single-digit numbers. Sometimes people may go up to addition and subtraction within 20 (10+10, 20-10) and for multiplication & division some may go up to multiplication with products up to 12 (12x12).



Grade	Standards
K	•Fluently +/- within 5.
1	•Fluently +/- within 10, have strategies for problems within 20.
2	•Fluently +/- within 20, know from memory within 18. •Fluently +/- within 100 using strategies.
3	•Fluently add/subtract within 1000 using strategies. •Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
4	•Fluently add/subtract multi-digit whole numbers using the standard algorithm.
5	•Fluently multiply multi-digit whole numbers using the standard algorithm.
6	•Fluently divide multi-digit whole numbers using the standard algorithm. •Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

CCSS

Not everyone uses the same standards so again I've pulled the standards from the CCSS. They state that in 2nd grade kids need to 'know from memory' facts within 18, which means up to 9+9, and in 3rd grade kids need to 'know from memory' products of two one-digit numbers, so up to 9x9. If you notice in those same standards it does say kids need to be fluent up to 20 (10+10) for addition & subtraction and up to 100 (10x10) for multiplication & division. So for this starter kit we are going to use that to

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define our “math facts”: doing any operation (add, subtract, multiply, divide) using just the numbers 0-10.

In the previous section of this kit we already discussed what ‘fluently’ entails (fast, accurate, & flexible) but I’d like to dive in a bit more into what ‘know from memory’ means. Let’s do an example exercise. I’d like you to memorize this 11 digit number:

25811141720

Do what you need to for a few moments to help you memorize that number.

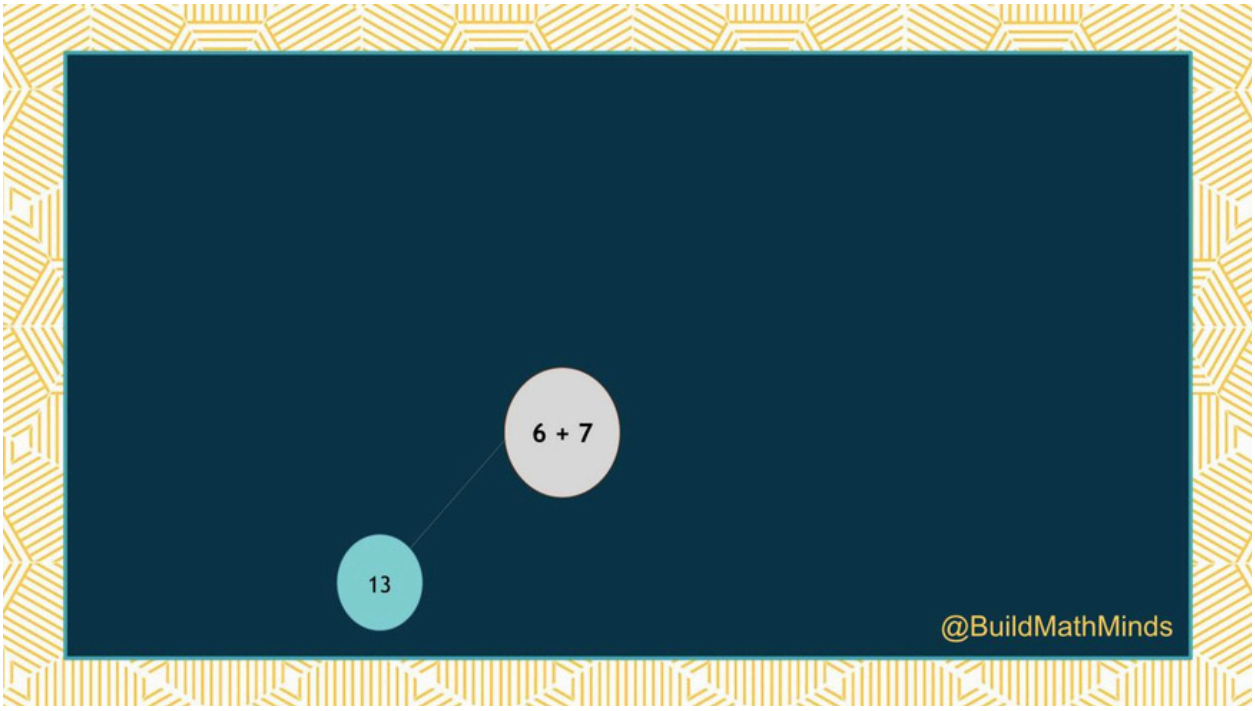
Seriously, stop and memorize it! Don’t just skip ahead.

So, what kinds of things did you do to help you memorize? Did you repeat itover and over? Maybe even say it out loud a few times? Did you write it down repeatedly? Did you have someone quiz you on it? Maybe you even created a flashcard of it?

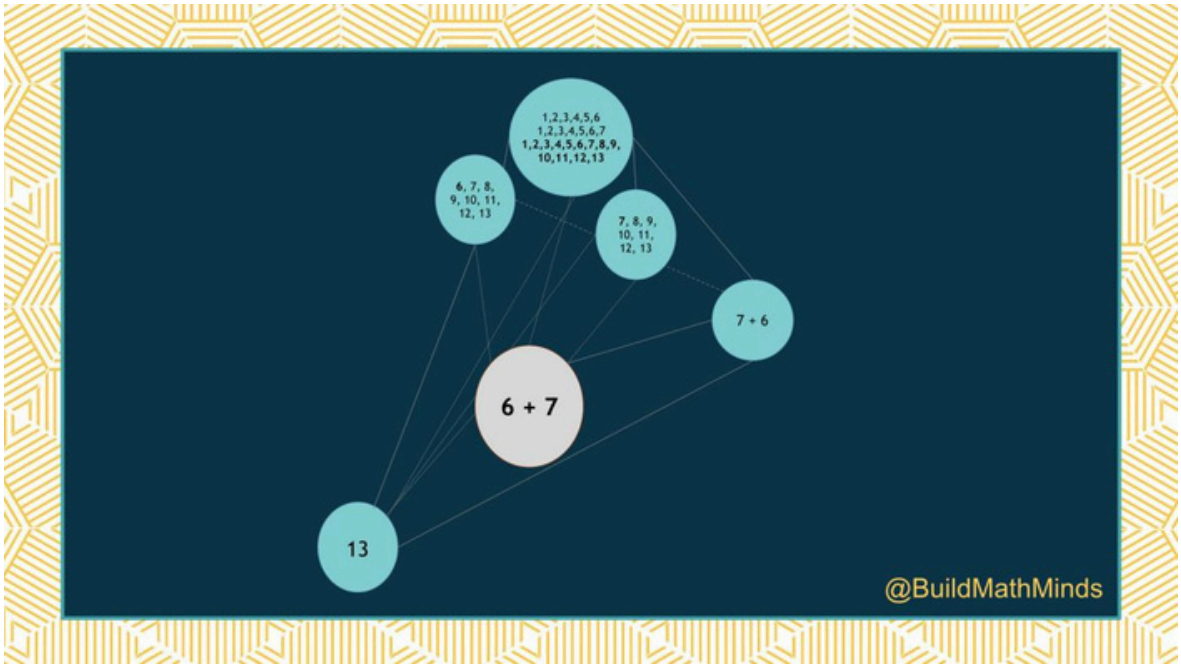
My guess is yes, you did. Maybe you only did a few of those things, or maybe you did all of them. That’s the way most of us were taught to memorize things we need to learn.

The problem with memorization is that it’s a short term solution. We can all memorize things when we are asked. We drill it into our brains and then usually as soon as we have used that information (like on a test of all the state capitals in the U.S.) we forget it very easily. This is due to a couple of things. One is the “if you don’t use it you lose it” idea. Which is a valid point. If we aren’t using those things repeatedly our brain decides to dump it to make room for other things that we do need to use. The other is that it’s very difficult to retrieve information that only has one connection point in your brain.

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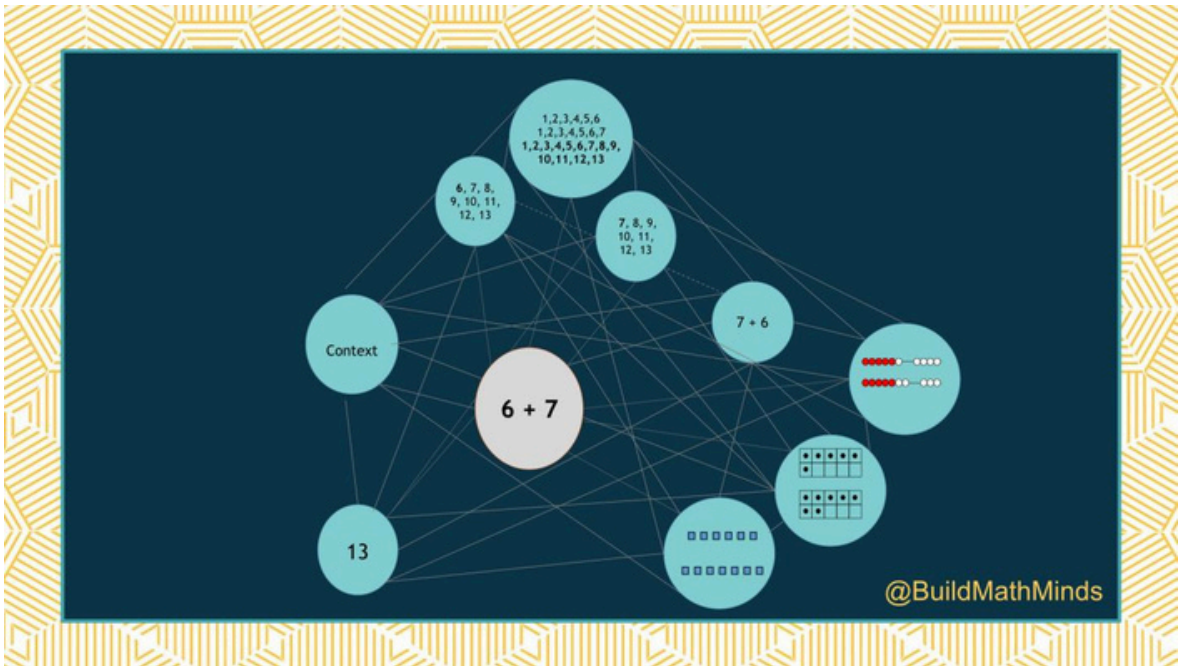


So when we have children memorize the fact of 6 + 7 = 13, it is only one connection point in their brain. If they can't retrieve it quickly (i.e. that connection point is broken), what is their fall back?
This is where the counting strategies come in that we talked about briefly in the first section of this kit. Kids directly model the problem, they count on, and they even start to see the connection between 6+7 and 7+6 and even 13-6 and 13-7.

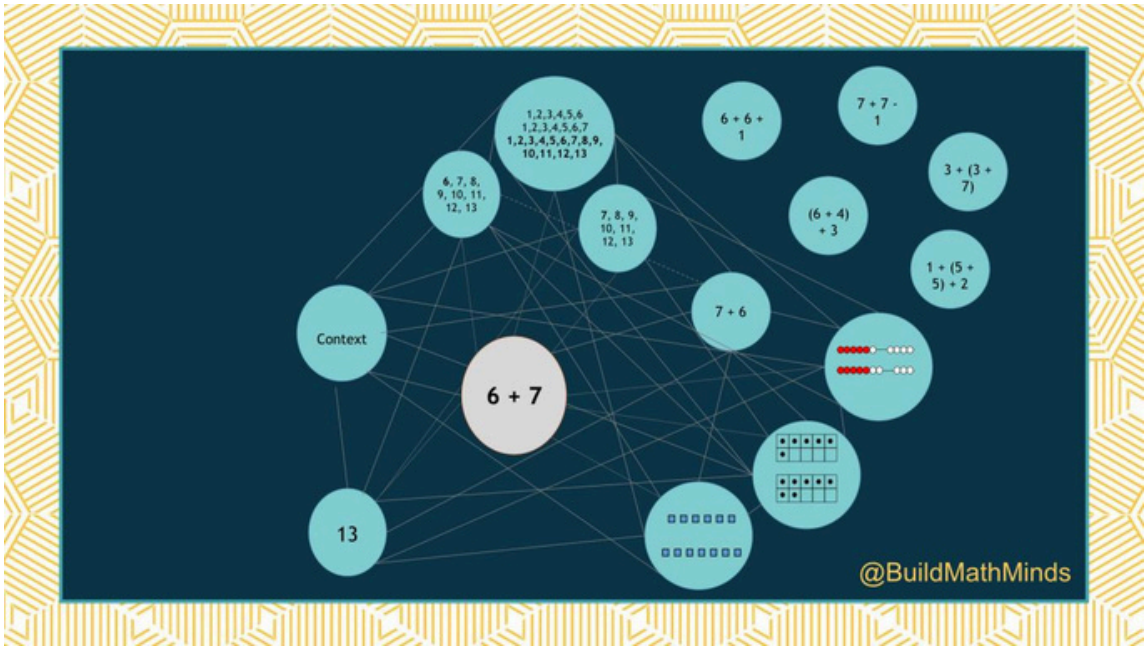


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This is a stage that moves students towards fluency. It gives them some flexibility that if they don't just instantly know the answer, they can figure it out using a counting strategy. So connections give students more flexibility and help them with their accuracy, but counting strategies are not very efficient. Teachers will often bring in lots of visual representations to help kids.

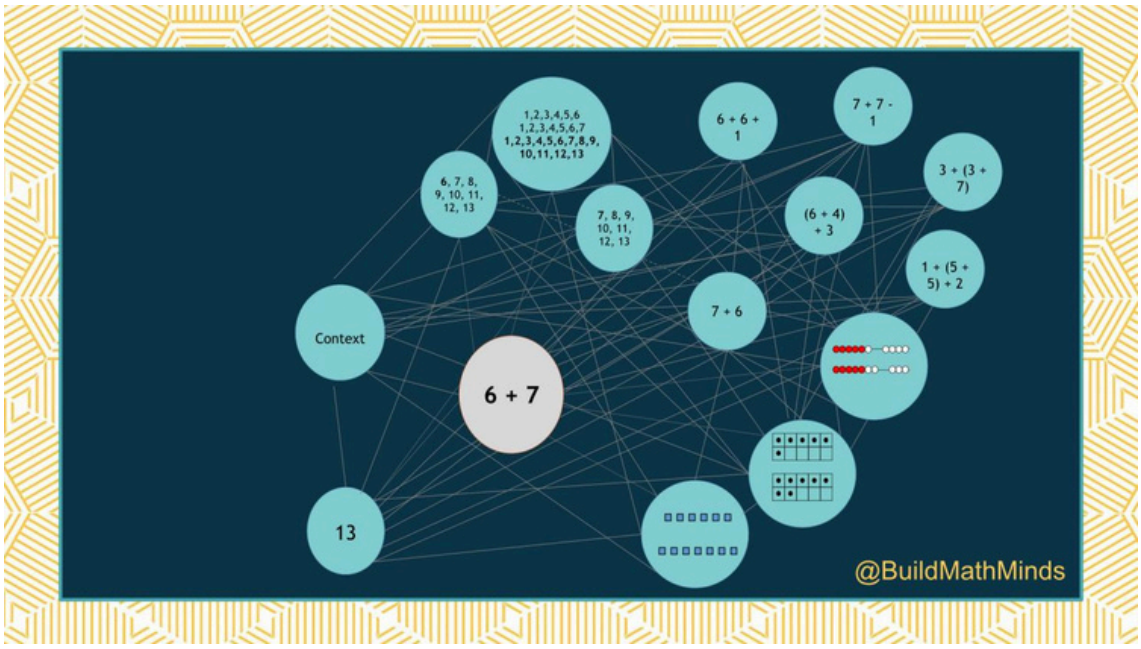


Ten frames, rekenreks, and individualobjects give kids the opportunity to count to solve a problem but can alsohelp kids get to the Derived Fact stage.



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Derived Facts are when kids use facts they know to connect to a fact they don't know. So for 6+7, if they don't instantly know it is 13 we would love for them to say something like "6 + 6 is 12 and 1 more makes 13." The problem is that often textbooks try to teach these Derived Fact strategies individually and students aren't making connections to the other things they know about 6+7. See how in the visual above there are no connections created? But this visual below shows what happens when these strategies are developed in a way that helps kids see relationships instead of isolated strategies.



The more connectionswehavebuiltaroundthingstheeasieritistorecall that information.Thisisthedifferencebetween*memorization*and*know from memory*.Whenyoumemorize,thereistypicallyjustoneconnection point. You'vememorizedthat6+7=13.Butwhenyouknowfrommemory that 6+7=13,youmightuseoneoftheseconnectionstohelpyouinstantly recall thatinformation.Nowremembertheultimategoalisstilltogetkids to instantlyknowtheirfacts.Helpingkidsdevelopttheseconnectionshelps make theconnectionbetween6+7and13evenstronger.

The mostimportantpiece of thisdifferenceisthatifaconnectionpoint breaks, youhaveotherconnectionpointstohelpyoubeabletorecallthat information.Evenifakid knowstheanswerinstantly(directconnection from 6+7 to13)theyhave theotherknowledgetofallbackoniftheyneed it. Now noteverychildwill haveallttheseconnectionsbuiltforjustthefact 6+7, but themoreconnectionstheydohavetheeasieritwillbeforthemto know thosefactsfrommemory.

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So let's revisit your 11 digit number. STOP! Don't go back and look at it. Can you remember the 11 digits that you memorized? Write it down here:

_____

Did you get it? Did yourmemorization work? For many people it does….for right now. Butif I asked you in two months what that 11 digit number is, most peoplewon't remember it.

Go to the next page andlet's take a look at those 11 digits in a different way.

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I'm going to space out the digits in a way that might help you build some connections.

2 5 8 11 14 17 20

Write down three things you notice about those numbers:

1._____

2._____

3._____

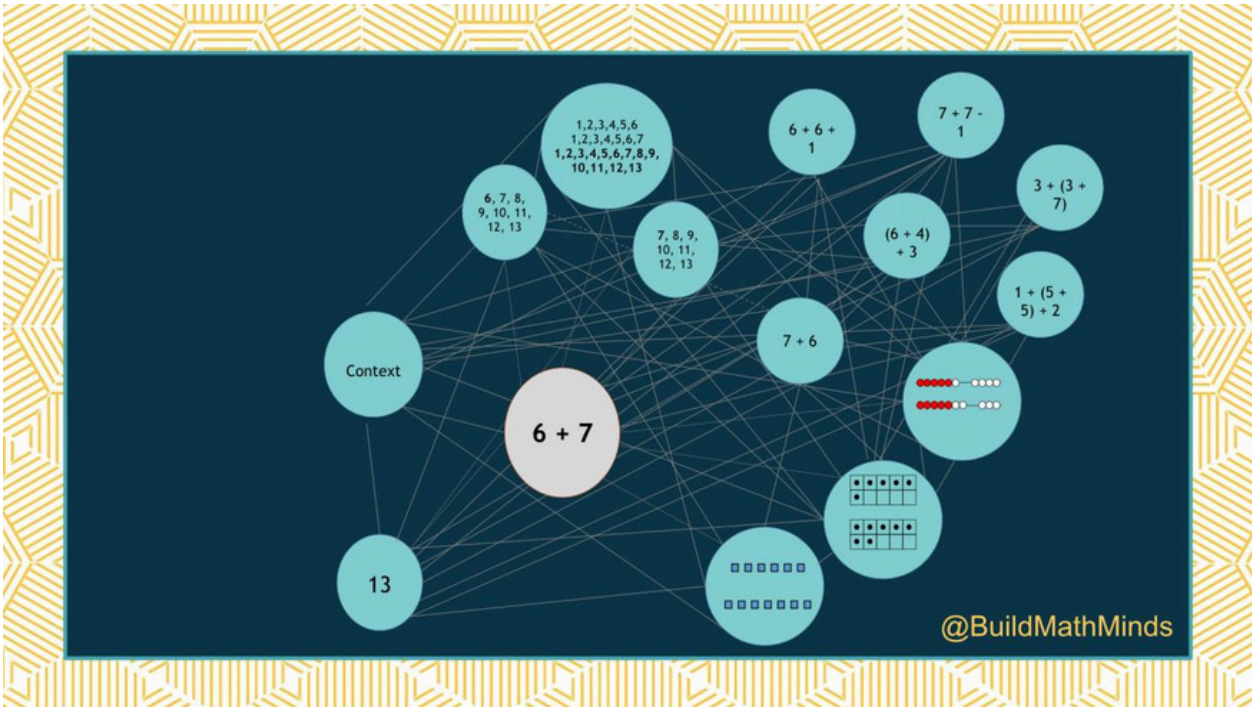
Those three things you noticed are connections that will help you 'know from memory' those 11 digits even if I came back and asked you in two months. Because now you aren't trying to remember 11 individual numbers and the sequence they were in. Now all you need to remember are those few connections.

1. The first number is 2
2. Add a 3 to the number
3. Stop adding 3 when you get another 2 (with a zero behind it)

Those were my connections. Some of you may have different connections. Some of you may have more than 3 connections. If you were born in February, the connection that the first number is the month you were born in will help you recall that. If you have 2 kids, that connection helps you remember the first number. The more connections you have, the easier it will be to recall those 11 digits, in the correct order, later.

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So what are those connections when it comes to helping our kids learn their math facts. I showed you this one example of 6+7



This may seem very overwhelming to think about teaching students all this for every one of the 121 Addition facts. But here's the thing: YOU DO NOT TEACH THESE.

You've probably already seen it happen in your classroom. You spend weeks (sometimes a month) trying to teach kids strategies like doubles and Make a 10, only to see them still counting on their fingers. WHY!?!?!?

Because strategies are personal. They get developed when kids have built connections between numbers. We can't expect a student to use the strategy of 6+6+1 or 7+3+3 to solve 6+7 if they don't first have a solid foundation of number sense built.

Howden (1989) said number sense is "...good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms."

I love the last part of the quote about how it develops, but the first part about number sense being a "good intuition" always frustrated me. It gave me no guidance as to what needed to be built for my students.

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What is it that children who have this 'good intuition' understand about numbers compared to children who do not have it? After years of trying to figure it out, I found the answer in John Van de Walle's book Teaching Student Centered Mathematics K-3 (2006). I originally read the book in 2007 (there is a new version out in 2017) and when I came across these four number relationships he mentions in the book a lightbulb came on for me:

1. **Spatial relationships** – having a visual of a quantity. Instead of seeing 3 as just the digit 3, kids need to see three things.
2. **One and two more, one and two less** – this is not the ability to count on two or count back two, but instead knowing which numbers are one and two less or more than any given number.
3. **Benchmarks of 5 and 10** – ten plays such an important role in our number system (and two fives make up 10), children must know how numbers relate to 5 and 10.
4. **Part-Part-Whole** – seeing a number as being made up of two or more parts.

At first I did not understand the full value of these four relationships and especially why we needed to develop them with small numbers. It was not until I started working with PreK-2 children around these four relationships that I realized this was the number sense that some kids were lacking.

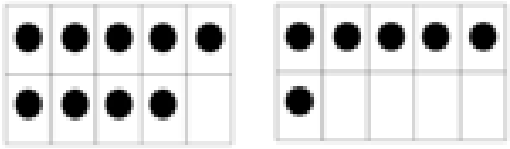
These relationships may seem trivial, even unimportant. However, if children do not have these four understandings around small numbers it affects their flexibility, and without flexibility they will not become truly fluent. The more I've worked with kids of all ages, these four relationships need to be built around all types of numbers (whole numbers, fractions, decimals, etc.). Let's take a look at how these relationships impact how a child thinks about single-digit addition, because remember that's where it all starts.

Think about the problem 9 + 6. A common strategy on that problem is thinking of it as 10 + 6 then subtract 1. For children who do not have number sense they cannot begin to follow the logic of the child who came up with this strategy. So, let's take a brief look at how those four relationships play a role in helping children develop the understanding that 9 + 6 = (10 + 6) - 1.

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Spatial Relationship

Spatial relationships help give children a visual to go along with a number instead of just the abstract numeral. Common visuals are dot patterns and ten frames, so a child who has this visual using ten frames in their mind for 9+ 6 has a better chance of seeing how they can take one from the 6 to create a 10:



One/Two More and Less

The spatial relationship, asseenin the ten frames above, help build the idea of One/Two More and Lessinthis problem because it gives children a concrete way to see that 9isjustone less than 10 because the ten frame is missing one.

Benchmarks of 5 and 10

If the objects are placed into the ten frame “five-wise’ (as they are in the picture above), it helps children develop an understanding of how that number relates to 5. No matter how objects are placed in a ten frame, the visual created helps children see how that amount relates to the benchmark of 10. This aids in helping children understand and use strategies based on using 10, because they are able to see that if the ten frame was full it would create the problem 10 + 6, but they need to take one away because it only had 9 instead of the full ten.

Part-Part-Whole

For too many children, 6 is just 6. They can not see how it can be broken down into parts that create the whole of six. Without that understanding, children will not decompose (break apart) numbers to help them make friendlier numbers. Some children may like creating a 10 when one of the addends is 9 but they do not like having to subtract one at the end. So, children often think about 9 + 6 as (9 + 1) + 5. The ability to use that strategy efficiently is dependent on a lot of these four relationships but without the understanding that the 6 can be decomposed into a 5 and a 1, the strategy would never even come to mind.

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So why are these relationships so important to develop before we expect children to develop the strategies? We cannot expect students to jump into thinking about numbers in these different ways until they develop number sense. As Ann Dowker (1992) states, "To the person without number sense, arithmetic is a bewildering territory in which any deviation from the known path may rapidly lead to being totally lost." (p.52)

It is important that children develop a strong understanding of numbers and their relationships before we ask them to compute. If they do not have that strong foundation, the only approaches they will have for addition and multiplication are counting on their fingers or rote memorization. Neither of those are acceptable. We want children to have a foundation of number sense which allows them to recall and transfer information with ease. That does not happen if we move to memorization and drill too quickly.
Brownell and Chazal (1935) found that if we move to an emphasis on speed too soon it just encourages children to become faster at their informal approaches (i.e. counting on their fingers). During the drill, they are just practicing their inefficient and sometimes inaccurate strategies, which leads us to think we need to drill them more.

However, Rathmell (1978) showed that developing children's derived fact strategies lead to better retention of their basic facts. So the goal is to facilitate children's development of derived facts, but we can not expect them to develop those if they first do not have number sense. Your textbook probably has lessons on Doubles and Make a 10, but if children do not have the four relationships Van de Walle (2006) discusses, which build a child's number sense, then those problems in the book just become tasks to complete, not a strategy that will be used long term.

Baroody (2009) provides evidence that a child's weak number sense often is due to their inadequate informal knowledge about numbers. We have children whose informal experience with numbers differ widely before they even come to school. If we spend time in the early grades letting children explore and develop their informal knowledge, it will build a child's number sense and increase their flexibility when working with numbers, then fluency will actually occur faster. Plus, by developing a sense of the relationships between numbers, children will be able to transfer that number sense when working with multi-digit numbers, fractions, and decimals as well.

"Number Sense Can't Be Taught, It's Caught."

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So a reminder, we cannot directly teach these strategies. We cannot directly teach fluency. It comes from kids building connections around numbers, i.e. number sense. Connections cannot be taught to kids. Number Sense can't be taught, it's caught. They need to experience and build those connections and number sense relationships themselves. We can't directly teach them, but we can provide the experience to help build them. That's what we will take a look at in Section 3 of this starter kit.

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3 Things to Help You Build Fluency with Math Facts

#1: Focus on Building Number Sense

So much of what we ask kids to do in mathematics is dependent upon their knowledge of numbers. If they do not have a strong foundation of number sense built, then they are reliant upon memorization and following procedures without understanding. Taking time each day to do a number sense activity will pay off because you will spend less time teaching strategies.

How to put this into action: There are lots of number sense activities out there. Google it and you'll find a ton. But the best ones are ones that help build the 4 Number Sense Relationships mentioned in this section. Ensure kids are seeing visuals, not just the symbols. Focus on building One/Two More & Less. The Benchmarks of 5 and 10 are important for numbers up to 20 but even beyond (and below), there are still benchmark numbers. For fractions, have kids focus on how the numbers relate to ½ and 1. As you are doing activities with any kinds of numbers, encourage kids to talk about the relationship to a nearby benchmark number. Part-Part-Whole activities encourage kids to break apart (and put back together) quantities. Another essential piece is having kids discuss and investigate WHEN they should break a number apart.

#2: Help Kids See Types of Facts

There are certain types of facts that kids learn first for addition and multiplication. These are kind of like 'power facts' because they are foundational and help kids advance into derived facts. For example, if you want kids to see how they can use x2 to help them figure out x4, they first must know what x2 is. So focus on helping kids become solid on these 'power facts' and the four relationships.

How to put this into action: Below are two images that show the addition and multiplication facts. The darker colored ones are the 'power facts' that we should focus on first. The lighter color of those are ones that kids can derive by using their number sense relationships. For example if a kid knows 5x8=40, they can use that to help them with 6x8 because it's just one more group of 8. Check to see if your students know the dark colored facts first. If they don't, then you've got your starting point. If they do know those facts, do some number sense activities to help them see relationships between the facts they know and the facts they don't.

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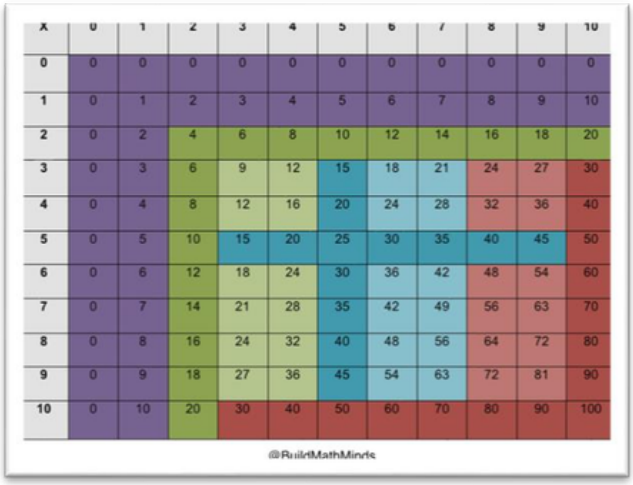
Addition Facts to focus on: Orange-Doubles, Green-Equal 10, Blue-Ten +something, Purple- Adding Zero



+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

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Multiplication Facts to focus on: Green-x2, Red-x10, Blue-x5, Purple-x0 and x1



X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

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#3: Assess Without Timed Tests

As mentionedinthissectionofthekit,researchhasshownthattimed tests do not helpchildrenadvancetomorefluentstrategies.However,we do still wanttoassesskidstoensuretheyarebecomingfluent.Spending time observinghowstudentsaresolvingproblemscantellyoualotabout their fluency.Youcanalsodoquickassessmentsofthetypesoffactswith individualstudentsandhavethemcolorintheirownfactchartshowing the ones theyknowandhavethemworkonseeinghowtheonestheydon't know areconnectedtotheonestheyknow.

How toputthisintoaction: OneofthefirstthingsIdidafterlearning about theCGIstageswastomakeachartsimilartotheonebelowthat I kept onaclipboard.AsIheardkidsdescribehowtheyknewananswer I'd put a checkmarkinthatarea.Overtime,ifthemajorityofcheckmarks are in the CountingOnphase,thenIwouldsaythatstudentwasinthat phase of theirfluencydevelopment.

	Direct Modeling	Counting On/Back	DerivedFact	Fact
Student A				
Student B				
Student C				

Recently, the book Math Running Records by Dr. Nicki Newton came out. This book details out how to assess all three parts of fluency for facts. You can download the assessments for free at https://www.mathrunningrecords.com/. The assessment takes a bit of time to do because it needs to be done one-on-one and with the teacher so that you can get information about your students' thinking. But the information is very valuable. You can choose to do assessments that are quick & easy, but don't give you information that informs your instruction or you can use an assessment that is lengthier, but gives you amazing information about how to help move your students forward along their path to fluency.

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Decision #3

Arranging YourMathTimeToMakeRoomFor Fluency

One thing that is just as important as looking at WHY fluency is so important is to spend some time thinking about WHAT it will look and sound like in your classroom when your students have developed their math fluency.

Yes, what you do in math is to help kids learn the standards, the actual math content, but what that looks like and sounds like can be very different in classrooms. What are things that you would love to hear your students say? What would it feel like in your room if students were operating with fluency? How would students approach problems?

Going forward, it will be hard to slow down and spend time focusing on building fluency through understanding instead of memorization and following procedures. So I really want you to imagine what it's going to be like when your students are showing up during math time with true fluency. These will help you stay positive and moving in the right direction when you start to wonder if what you are doing is really making a difference.

Examples to Inspire You:

- ❖ Students are able to accurately and efficiently solve problems.
- ❖ Students are flexible with strategies and can choose which strategy to use based upon the context of the problem and the numbers within the problem.
- ❖ Students are attempting problems they aren't sure how to solve instead of waiting for me to show them how to solve it.
- ❖ Students are sharing and comparing their strategies.
- ❖ Students see their mistakes or misconceptions as steps along their path to fluency, not that they are bad at math.
- ❖ All students have access to the math tasks. Even if they can't solve it the way the book says, they know they can solve it in a way that makes sense to them.
- ❖ Students freely talk about their thinking, even when they aren't sure it's correct. Students see their sharing as a way to clarify their thinking, not just prove they have a correct answer.

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Now it's your turn. Grab your notebook and write down what you want your students to be like as they start to operate with fluency in mathematics. What are the most important things that you want to help your students build? Keeping a reminder of these things will help you stay focused on building that fluency when things aren't going so great. So don't skip this step!

After you've written down your ideas of what operating with fluency will look like in your students, take time to make an anchor chart of those items and post it in your classroom. It will help remind you and your students what your expectations are of them as mathematicians.

How to Build Mathematical Fluency in Your Classroom

The final step towards building your students' fluency is deciding HOW you are going to implement these ideas into your classroom. I am not going to detail out exactly how you should do that. That's not what I believe we should do for teachers just like I don't believe that's what we should do for our students in math. We should NOT make everyone do it the same way. Every classroom is filled with kids of varying needs and abilities. Every classroom has a teacher with their own teaching style. So what works for one classroom will not always work for everyone else. I understand that having some kind of guideline is important.

Everyone's situations are different. You may be tied to a textbook or you might be in a district that doesn't use a textbook and the standards are your only guide. No matter your situation there are three types of activities I recommend trying to incorporate as much as you can.

1. Number Sense Routine
2. Story Problems
3. Games

We go into more ideas and detail in my Flexibility Formula course on all three of these but here is where the fun part happens (if you remember at the beginning of this kit I said this decision is the fun part).

It's all about meeting your students where they are right now. What areas of number sense do your kids need to develop? How can you craft story problems that highlight your students and their interests? What games can you use to help give kids practice and repetition?

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These are all questions that have answers specific to you, your students, their culture & community, your teaching style, and your students' learning styles. I'm not going to tell you to use a game that uses tons of dice and gets kids excited and hollering during the game. Because not every teacher can handle that. If the make-up of the activity drives you crazy you will never use it.

So we will end this starter kit with where you start. I want you to start by thinking about how much time you've spent focusing on building fluency…true fluency…not just helping kids get fast & accurate, but their flexibility as well. Remember, we've all been there. We've all taught kids just to be quick and correct. But as we learn better we do better. Don't spend time thinking about all the ways you've taught math wrong in the past, focus on how you can change for the kids in the future.

There is so much we could do, but as you start to think about ways to adapt the lessons you've done in the past I want you to focus on those three types of activities to provide your students: Number Sense, Story Problems, and Games. Maybe you only pick one of those to focus on for right now. Maybe you are able to do all three. Do what works for you, just like I've recommended in this kit for you to do for your students.

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Resources for Incorporating the 3 Types of Math Activities

#1: Number Sense

The power of number sense activities is not all children see it one way. The activities provide opportunities for children to express how they see it and make connections to how others see it. Many number sense activities can be done when you have a spare two minutes as you wait to go to music. But I would encourage you to see where you can squeeze 5-10 minutes to fit a number sense activity each day. Some activities take more time, but could be used as math centers after doing them as a whole class. Whatever the type of activity, or amount of time they take, the more you do them the better children will get at developing the relationships.

Resources to put this into action:

Teaching Student-Centered Mathematics Prek-2 by Van de Walle, et al
Number Sense Routines books by Jessica Shumway (K-3 and 3-5)
My online Flexibility Formula courses for elementary teachers
Online Curation of Daily Number Sense Routines by Ryan Dent

#2: Story Problems

Story problems help kids see mathematics in contextual, real-life scenarios. One of the problems is that story problems in textbooks are not written for your students. What is 'real-life' for one group of kids is not the same for another group of kids. So, it's often best to create story problems that relate to your students. You can use the resources listed below as a reference for the different types of story problems and modify the problems to be contexts and numbers that your students can relate to. Typically, story problems are saved for the end of a lesson to show kids how what they just learned relates to their real life. Try starting your lesson with a story problem so they make that connection from the very beginning.

Resources to put this into action:

Greg Tang's Word Problem Generator
Addition and Subtraction Problem Type Chart
Multiplication and Division Problem Type Chart

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#3: Games

Kids need practice, but usually math practice is done in a boring and monotonous way through worksheets and drill. Games are a great alternative to this because kids will practice a lot of math problems in a 5 minute game yet in a fun and engaging way. Try to find 5-10 minutes as often as you can to let kids play a game to practice. This does not need to be during math time. If you end up with five minutes before lunch, pull a game out and let them play.

Resources to put this into action:

Video about using Games for Math Practice
Well Played books
NRICH compilation of games

Make sure to watch the Building Math Fluency video series https://buildmathminds.com/building-math-fluency for more in-depth information about building math fluency in the elementary grades.

The resources below are mentioned in the video series.

- Building Math Fluency Through Games
- Math Running Records video
- Howard County's website (click on your grade level and then browse around the 'Course Essentials')
- Number Sense Routines books: K-3rd and 3rd-5th
- Choral Counting & Counting Collections Book
- Steve Wyborney's site
- Berkeley Everett's Math Flips Flashcards

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Teaching math is not easy. It seems even harder now that we are being asked to teach math differently than we learned it. But the best way to "teach" math is not to teach it at all. Instead, listen to your students' mathematical thinking that they already have and use that to decide what type of Number Sense Routine you could use to help them progress towards the standards.

Look to your students to decide what number sets and context to use for a Story Problem and let them actually solve the problem using their own strategies. Then have them Practice every day to help solidify the current and past concepts through a game.

I believe educators have been bombarded with too many things to try in their classroom. So much so that we get paralyzed by all the choices.

Now I know that you may not be able to throw your textbook out the window and do these 3 things.

Use these as a basis of what your students need to experience around mathematics and look at how they can fit into what is already working for your students...or how can what is working now fit in with these 3 things????

Christina

P.S. I'd love to see here what you've learned in this kit. Post your big takeaways from this kit and tag me @buildmathminds.

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Your Plan of Action

What can you commit to doing, Perfectly Imperfect?



Implementing new things is often scary because we don't want to get it wrong. But I want to encourage you to just try. It most likely won't work great the first time. Take small steps, it doesn't have to be perfect but you do need to start.

It's also helpful to find a colleague to try things out with. It helps if you aren't the only person in your school trying to change how you build students' fluency.

Using the information from the kit and the ideas given on implementation, answer these questions:

What is ONE 'A-HA' you had while reading this fluency kit?

_____

_____

_____

_____

What is ONE thing you are going to do to help your students build fluency?

_____

_____

_____

_____

What is ONE thing you are going to share with a colleague?

_____

_____

_____

_____

_____

One last question, what's that 11-digit number? Did you use the connections you built or memorization? Most likely it was the connections, just like your students will.

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